

Calculators and Mobile are not allowed

Answer all of the following questions

1. Find the following limit, if it exists:

$$\lim_{x \rightarrow 0} \frac{x^2 + x \sin x}{x \tan(2x)} \quad (3 \text{ pts})$$

2. Let

$$f(x) = \begin{cases} \frac{x\sqrt{x^2+1}}{|x|} + \sin x, & \text{if } x < 0, \\ 3x^3 - x^2 + A, & \text{if } x \geq 0. \end{cases}$$

Find the value of A so that f is continuous for all real numbers. (4 pts)

3. Find the x -coordinates of all the points on the graph of

$$f(x) = 3x - 2\cos x, \quad 0 \leq x \leq 2\pi$$

at which the tangent line is perpendicular to the line $x + 4y = 5$. (3 pts)

4. If

$$f(x) = \sqrt{\sin x} + \sin \sqrt{x},$$

find $f'(x)$. (4 pts)

5. An isosceles triangle has equal sides 10 cm long. If the area of the triangle is decreasing at a rate of $2 \text{ cm}^2/\text{sec}$. Find the rate of change of the angle θ between the equal sides of the triangle, when $\theta = 60^\circ$. (4 pts)

6. Does the graph of the function

$$f(x) = x^{\frac{1}{3}}(4-x),$$

have a cusp, a vertical tangent line? Explain. (4 pts)

7. Find an equation of the tangent line to the graph of

$$x^3 y + y^3 x = 10,$$

at the point $P(1,2)$. (3 pts)